Relative Injustice Aversion

Luis José Blas Moreno
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Luis José Blas Moreno Garrido
Departamento de Métodos Cuántitavivos y Teoría Económica
Universidad de Alicante
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Abstract
I propose a new utility function based on the relative aversion to injustice to explain why, in classical bargaining games, classical equilibria do not hold when money is not windfall, but it is result of the effort.

JEL classification: D3 - Distribution
D63 - Equity, Justice, Inequality, and Other Normative Criteria and Measurement
D64 - Altruism
P14 - Property Rights

1 Introduction

Subjects’ behavior in the lab in bargaining games differs from the theoretical prediction suggested by classical models. The mean donation by participants in the dictator game in the lab is about 20% of the endowment (see Camerer, 2003 and Engel, 2011 for a review).\footnote{For a review of bargaining games such as dictator game or ultimatum game, see section 2.}

This result seems robust in some circumstances. For instance, when the size of the pie is doubled (Forsythe et al., 1994), when dictator’s decision is anonymous even for experimenters (Hoffman et al., 1994)\footnote{Eckel and Grossman (1996) claim that even with anonymous conditions, dictator may increase her offer when she knows the characteristics of the recipient (for instance, when recipient is an established charity, offer triples with respect a recipient from the same population as dictator).} or even when subjects come from different cultures (Henrich et al., 2001).

Some models try to explain this behavior with the introduction of fairness or social preferences. For instance, Fehr and Smith (1999) (hereafter FS) introduce the concept of inequality aversion. Other explanations for this deviation of theoretical prediction are social pressure, guilt, sympathy (see Andreoni, 1990), confusion (Andreoni, 1995),\footnote{Andreoni distinguishes between kindness and confusion. He claims that half of collaborations in an experiment came from people who understood the free-riding opportunity, but chose to cooperate.} coop-
eration (Fehr and Gachter, 2000), altruism (Fehr and Gachter, 2002 and Andreoni and Miller, 2002), reputation (Milinski, 2002 and Nowak et al., 2000) and so on. The sense of fairness seems to be common in the whole mankind (Henrich, et al., 2001), however, the “size” of this fairness varies among cultures (Henrich, 2000). Altruistic preferences have been study from a biological point of view. Brañas et al. (2013) find that the exposure to prenatal sex hormones affects altruism. Although genes affects altruistic behavior and risk aversion, genetic differences only explain approximately twenty percent of individual variation (Cesarini et al., 2009).

However, it has been shown that the percentage given by dictators is context dependent, especially when money is not manna for Heaven, but it is the result of an agent’s effort. Cherry et al. (2002) show that when the size of the pie is determined by dictators’ effort, under anonymous conditions, 95% of dictators give nothing. However, when the size of the pie is determined by recipients’ effort, dictators will be willing to give away a larger percentage of money (Ruffle, 1998 and Oxoby and Spraggon, 2008).

The idea of deserving is also presented in experiental studies such as Frohlich, Oppenheimer and Kurkiet (2004) (hereafter FOK) and Rodriguez-Lara and Moreno-Garrido (2012a). Current behavior models are not able to explain rejections in bargaining games with non-zero sum. The inequality aversion model of FS punishes deviations from fifty percent of the pie. However, when money is the result of an effort, agents could believe that what they deserve is different than the half of the pie. Extensions of FS model are proposed by FOK and Rodriguez-Lara and Moreno-Garrido (2012b), but those models allow contradictory feelings such as envy and guilty simultaneously. This paper proposes a new theoretical model for bargaining games with production that incorporates relative injustice aversion. In this model, agents suffer from inequality, when the injustice committed against her (defined as the difference between what she thinks she deserves and the material payoff) is different of the injustice committed against her counterpart. Theoretical predictions of several bargaining games are computed as a function of the parameters of the model. Bargaining power, defined as the difference between the proposer profit and her deserved, is computed in each bargaining game.

The rest of the paper is organized as follows. In section 2, I review two classical bargaining games and modifications of them. The theoretical model is presented in section 3 and in section 4 theoretical equilibria of presented bargaining games are computed. Finally, section 6 concludes.

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4 Although cooperation it is observed in several animal species (Dugatkin, 1997), fairness it is observed only in a few species. For a long time, it was thought that justice was the product of human evolution. Examples of inequality aversion have been found in Capuchin monkeys (Brosnan and Waal 2003) whose common ancestor with man has around 50 million years. However, in a recent study, Jensen et al (2007) claim that chimpanzees (whose common ancestor with man has only 7 million years) do not show fairness. It suggests that fairness is not only an evolutionary issue, but it depends on social structure of individuals and environment.

5 Technically, the dictator game is a constant sum game. However, without loss of generality, the payoff can be normalized, and it can be supposed that dictator game is a zero-sum game. Ultimatum game is not a zero-sum game because the sum of payoff is different, depending on whether responder accepts or rejects the offer.
2 Bargaining Games

In this section, I review two classical bargaining games and modifications of them: dictator game, ultimatum game, $\delta$-ultimatum game, $(\delta_1,\delta_2)$-ultimatum game and $(0,1)$-ultimatum game.

The ultimatum game (Güth et al., 1982) is a two-player game, where one of them, called proposer, has to decide how to split a certain amount $M$ between herself and the second player, called responder. Suppose that proposer offers $x$ to responder, then the initial allocation is $(M-x,x)$. If responder accepts the offer, then it is carried out. However, if responder rejects the offer, then both players obtain nothing. Dictator game is a particular case of ultimatum game, where responder is forced to accept. In this game, the Nash equilibrium for the dictator, is to keep the whole pie. In the ultimatum game, in a continuous strategic space, the Subgame Perfect Nash equilibrium (SPNE) is that profile in which proposer offers nothing and recipient accepts any non-negative offer. The classical example of the ultimatum game is a bargaining between a seller and a buyer that have to decide how to split the surplus of a traded good.

The $\delta$-ultimatum game is a modified ultimatum game introduced by Suleimain (1996). It falls between the dictator game and the ultimatum game. In this game, like in the ultimatum game, proposer has to decide how to split a certain amount $M$ between herself and responder. If responder accepts the offer of $x$, then, the payoffs are also $(M-x,x)$. However, if responder rejects the offer, then payoffs are multiplied by $\delta$, and they are $(\delta(M-x),\delta x)$. Note that when $\delta = 0$, then the game matches the ultimatum game and when $\delta = 1$, rejection does not play any role and then, the game matches the dictator game. Since dictator and ultimatum game have the same theoretical prediction, then the $\delta$-ultimatum game should have the same prediction as well. However, as Suleiman proved, proposer’s offer increases when $\delta$ decreases, and therefore proposer is more afraid of a possible rejection, when the cost of it (proportional to $1-\delta$) increases.

Another bargaining game proposed by Suleiman is the $(\delta_1,\delta_2)$-ultimatum game. The unique difference from $\delta$-ultimatum game is that in case of rejection payoffs are $(\delta_1(M-x),\delta_2x)$. That is, proposer’s payoff is multiplied by $\delta_1$ and responder’s payoff is multiplied by $\delta_2$.

Finally, I propose the $(0,1)$-ultimatum game (hereafter $(0,1)$UG) as particular case of the $(\delta_1,\delta_2)$-ultimatum game, where $\delta_1 = 0$ and $\delta_2 = 1$. In this game, in case...
of rejection, proposer obtains nothing and responder keeps the initial proposal. Table 1 summarizes the payoffs of those bargaining games.

Since rejections in \((0, 1)\)UG does not affect responder’s payoff, the only reason that could motivate a rejection in this bargaining game are psychological emotions such as inequality aversion or injustice aversion. This game could be understood as a kind of trust game. Proposer has to put her trust in responder, and wait an acceptance if the offer is fair. An example of \((0, 1)\)UG could be the same bargaining game proposed in dictator game, with the proviso that buyer, after the trading is asked to fill out a survey and in case of negative report, the buyer loses her profit.

In this game, the SPNE is such that proposer offers the minimum amount that responder is willing to accept and responder accepts. However, if responder rejects any offer, then any initial allocation for proposer is part of a SPNE.

<table>
<thead>
<tr>
<th>Game</th>
<th>Acceptance</th>
<th>Rejection</th>
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<tr>
<td>Dictator Game</td>
<td>((M - x, x))</td>
<td>((M - x, x))</td>
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<tr>
<td>Ultimatum Game</td>
<td>((M - x, x))</td>
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<tr>
<td>(\delta)-Ultimatum Game</td>
<td>((M - x, x))</td>
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<td>((\delta_1, \delta_2))-Ultimatum Game</td>
<td>((M - x, x))</td>
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<tr>
<td>((0,1))UG</td>
<td>((M - x, x))</td>
<td>((0, x))</td>
</tr>
</tbody>
</table>

Table 1: Payoffs for an acceptance or rejection of an initial allocation \((M - x, x)\).

3 Theoretical model

Material utility function (also known as selfish utility function) only depends on monetary payoffs. However, some authors, as Rabin (1993), Fehr and Smith (1999) or Konow (2000) introduce psychological payoffs in the utility function. Let us define formally those utility functions.

**Definition 1** (Selfish utility function). Let \((S, P)\) a game with 2 players, where \(S = S_1 \times S_2\) is the strategy space and \(P = (p_1, p_2)\) are the material payoff function where \(p_i (s_1, s_2)\) depends on the strategy \((s_1, s_2) \in S_1 \times S_2\).

If the utility function of player \(i\) can be represented by an increasing function \(f\) of the payoffs \(p_i (s_i, s_j)\)

\[
u_i (s_i, s_j) = f (p_i (s_i, s_j)) \quad \forall s_i \in S_i \forall s_j \in S_j, \quad i, j \in \{1, 2\}, i \neq j
\]

then we say that the utility function \(u_i\) is *selfish*.

**Definition 2** (Subjective utility function). We say that the utility function \(u_i\) is *subjective* if it can be represented by

\[
u_i (s_i, s_j) = f (p_i (s_i, s_j)) + F(p_i (s_i, s_j), p_j (s_i, s_j), \theta)
\]
where \( F : \mathbb{R}^2 \times \Theta \to \mathbb{R} \), is a nonzero function that depend on the own payoff \( p_i (s_i, s_j) \) and other payoffs \( p_i (s_i, s_j) \) and a set of variables \( \Theta \).

The set \( \Theta \) represents psychological payoffs such as beliefs (Rabin, 1993), envy or guilty (FS) or fairness (Konow, 2000). FS introduced one of the pioneer subjective utility functions with the concept of inequality aversion. The FS utility function not only depends on the own payoff, but also on the differences of payoffs. For agent \( i \), her utility function is defined as

\[
\begin{align*}
\text{u}_i (s_i, s_j) &= p_i (s_i, s_j) - \alpha_i \max \{p_j (s_i, s_j) - p_i (s_i, s_j), 0\} \\
&\quad - \beta_i \max \{p_i (s_i, s_j) - p_j (s_i, s_j), 0\},
\end{align*}
\]

where the term associated to \( \alpha_i \) depicts the disadvantageous inequality aversion, i.e., the loss of utility that agent \( i \) has if agent \( j \) has a larger monetary payoff. This model also assumes that agent \( i \) suffers from disutility if agent \( j \) has a relative smaller monetary payoff. The term associated to \( \beta_i \) captures this advantageous inequality aversion. FS suppose that \( \alpha > \beta > 0 \).\(^{10}\) Hence, agents dislike inequality, but they dislike more the disadvantageous inequality aversion (envy) that the advantageous one (guilt).

Now, I analyze the dictator and the ultimatum game. Let \( M \) be the initial amount to split in the dictator game. When dictator offers \( x \) to recipient, material payoffs are \( p_1 (x, s_2) = M - x \) and \( p_2 (x, s_2) = x \). The solution for the linear model (1) is \( x = 0 \) if \( \beta_1 < 1/2 \) and \( x = M/2 \) if \( \beta_1 > 1/2 \).\(^{11}\) In other words, if the advantageous inequality aversion parameter (\( \beta_1 \)) is large enough, then dictator prefers share the half of the pie in order to avoid guilt associated to an unequal distribution.

In the ultimatum game, proposer not only considers the parameter \( \beta_1 \), but also the parameter \( \alpha_2 \), because an unfair offer may cause a rejection of responder. If proposer offers \( x \in [0, M/2] \),\(^{12}\) responder will accept, if the utility to accept the offer \( x \), \( u^a_2 (x) \), is greater than or equal to the the utility to reject \( x \), \( u^r_2 (x) \),

\[
\begin{align*}
\text{u}^a_2 (x) &= x - \alpha_2 (M - 2x) \\
\text{u}^r_2 (x) &= 0.
\end{align*}
\]

If \( x \geq \frac{\alpha_2}{\alpha_2 + 1} M \), then responder will accept the offer.\(^{13}\) However, if the proportion shared is smaller than \( \frac{\alpha_2}{\alpha_2 + 1} \), then responder will prefer reject the offer in order to reduce the disadvantageous inequality aversion. Figure 2 represents the responder’s utility when parameters \( \alpha_2 \) and \( \beta_2 \) change.

In the FS model, both agents believe that the fair allocation is the half of the pie for each one. However, in bargaining games with production, agents could think that what they

\(^{10}\)Some agents may like inequality (i.e., for whom \( \beta < 0 \)). In competitive games, \( \beta < 0 \) yields the same prediction that \( \beta = 0 \). Hereafter, I suppose that \( \beta \geq 0 \).

\(^{11}\)With a non-linear model, interior solutions in the interval \( [0, M/2] \) are reached.

\(^{12}\)Offers above \( M/2 \) are dominated.

\(^{13}\)In case of indifference, it is assumed that responder accepts because otherwise, there would not be equilibrium in a continuous strategy space.
Figure 2: Responder’s utility in the ultimatum game when she receives $x$ and the proposer keeps $M - x$, for $\alpha_2 = \beta_2 = 0.25$ (solid line), $\alpha_2 = \beta_2 = 0.50$ (dashed line) and $\alpha_2 = \beta_2 = 0.75$ (dotted line). Null payoff stands for a rejection. 45 degrees line represents the classical model ($\alpha_2 = \beta_2 = 0$).

deserve is not the equal split, but their contribution to it ($C_i$). In this line, FOK proposes the utility function\(^\text{14}\)

\[
\begin{align*}
    u_i(x_i, x_j) &= p_i(x_i, x_j) \\
    &- \alpha_i \max \{p_j(x_i, x_j) - p_i(x_i, x_j), 0\} \\
    &- \beta_i \max \{p_i(x_i, x_j) - p_j(x_i, x_j), 0\} \\
    &- \gamma_i \max \{C_i - p_i(x_i, x_j), 0\} - \psi_i \max \{p_i(x_i, x_j) - C_i, 0\}.
\end{align*}
\]

In equation (2), the term associated to $\gamma_i$ represents a disadvantageous injustice aversion that agent suffers when she receives less than her contribution. In the same way, the term associated to $\psi_i$ represents advantageous injustice aversion. This concept of injustice aversion is a generalization of FS model, when the game is a zero-sum game. In FS, an agent suffers from inequality when payoff moves away from the equal division and in the injustice aversion model, an agent suffers from injustice when payoff moves away from what she believes she deserves.

This utility function can explain offers over equal division when the contribution of recipient is greater than the contribution of proposer. However, when $\alpha_i$ and $\beta_i$ are zero (or very low), FOK utility function is not able to explain a rejection in the ultimatum game. Suppose that proposer offers $x_1 < C_2$, then, if $\alpha_2 = \beta_2 = 0$ is assumed, then responder’s

\(^{14}\text{For an extension of this model and discussion of justice principles see Rodriguez-Lara and Moreno-Garrido (2012b).}\)
utility for an acceptance

\[ u_2^1 (x) = x_1 - \gamma_2 (C_2 - x_1) , \]

and for a rejection

\[ u_2^2 (x_1) = -\gamma_2 C_2 , \]

and therefore, responder will always accept, thus this concept of injustice aversion does not explain rejections in the ultimatum game, because a rejection (both players obtain nothing) eliminates the inequality aversion in FS, but it does not eliminate the injustice aversion.

I wish to define a utility function that keeps the essence of injustice aversion, where it is defined a \textit{deserving point}, but it does not fail in non zero-sum games. I define the relative injustice aversion, that considers the difference between the injustice committed against, or for, both agents.

**Definition 3.** The utility function with relative injustice aversion is

\[
 u_i = (x_i, x_j) = p_i (x_i, x_j) - \lambda_i \max \{ (D_{ii} - p_i (x_i, x_j)) - (D_{ij} - p_j (x_i, x_j)), 0 \} - \mu_i \max \{ (D_{ij} - p_j (x_i, x_j)) - (D_{ii} - p_i (x_i, x_j)), 0 \},
\]

where \( D_{ij} \) stands for what agent \( i \) thinks that agent \( j \) deserves.

The term \( D_{ij} - p_j (x_i, x_j) \) is the injustice that agent \( i \) thinks that is committed against agent \( j \).\(^{15}\) The term

\[
 -\lambda_i \max \{ (D_{ii} - p_i (x_i, x_j)) - (D_{ij} - p_j (x_i, x_j)), 0 \}
\]

represents the disutility of being treated relatively worse than her counterpart (disadvantageous relative injustice aversion) and the term

\[
 -\mu_i \max \{ (D_{ij} - p_j (x_i, x_j)) - (D_{ii} - p_i (x_i, x_j)), 0 \}
\]

stands for the disutility of being treated relatively better than her counterpart (advantageous relative injustice aversion). As in FS, I also suppose that the advantageous relative injustice is less damaging than the disadvantageous relative injustice, i.e., \( \mu_i \leq \lambda_i \). Note that if agent \( i \) suffers from disadvantageous relative injustice aversion, then agent \( j \) suffers from advantageous relative injustice aversion in the same proportion. It is also supposed that parameters are bounded below; \( \lambda_i > 0 \) and \( \mu_i \geq 0 \).\(^{16}\)

Since it is supposed that agent \( i \) is rational, then the sum of what she thinks she deserves \( (D_{ii}) \) and what she thinks that her counterpart deserves \( (D_{ij}) \) must sum the total amount,

\(^{15}\)When \( i = j \), \( D_{ii} - p_i (x_i, x_j) \) is simply the injustice that agent \( i \) thinks is committed against herself. Note that it can be positive (advantageous) or negative (disadvantageous).

\(^{16}\)If \( \lambda_i = 0 \), then \( \mu_i = 0 \), and therefore we have the classical selfish utility function. The parameter \( \mu_i \), that represents the advantageous relative injustice aversion could be had bounded above by 1 (\( \mu_i < 1 \)), using a similar argument that FS use for the bounding for \( \beta_i \) in their model, but it is not necessary for the aim of this study.
i.e., $D_{ii} + D_{ij} = M$. Thus, the utility function can be rewritten with relative injustice aversion as

$$u_i(x_i,x_j) = p_i(x_i,x_j) - \lambda_i \max\{2D_{ii} - M + p_j(x_i,x_j) - p_i(x_i,x_j), 0\} - \mu_i \max\{M - 2D_{ii} + p_i(x_i,x_j) - p_j(x_i,x_j), 0\}.$$  

Note that if $D_{ii} = M/2$ (*egalitarian principle*) then the utility function matches up with inequality aversion utility function, making $\lambda_i = 2\alpha_i$ and $\mu_i = 2\beta_i$. FS use the half of the pie ($M/2$) as focal point, whereas, I consider this focal point $D_{ii}$ as a free parameter. In this way, it can take different values according to beliefs about property rights derived from production.¹⁷ For this reason, relative injustice aversion utility function generalizes the inequality aversion function of FS.

However, with this utility function, it is not possible to capture all the feelings that FOK utility function does. The latter allows contradictory feelings such that envy and guilty simultaneously. However, our utility function does not allow several focal points. Suppose, for instance, the case that

$$p_j(x_i,x_j) > p_i(x_i,x_j) > C_i,$$

that is, agent $i$ has less payoff that her counterpart, but her payoff is greater that her contribution, thus in FOK model (equation 2), she suffers from two-fold disutility. On the one hand, she suffers from disadvantageous inequality aversion ($\alpha_i$), because she obtains less than the half of the pie, and from another hand, she suffers from advantageous injustice aversion ($\psi_i$), because she obtains more than her contribution. Those disutility terms have both negative sign and hence they reduce the utility. However, with the relative injustice aversion utility function, agents either feel envy or feel guilty, that is, agents cannot have both feelings simultaneously.

## 4 Theoretical equilibria

In this section, the Subgame Perfect Nash Equilibrium is computed for each of the bargaining games presented above, using the relative injustice aversion utility function (3). I analyze the dictator game, the ultimatum game and the $(0,1)$-ultimatum game. Details are relegated to the Appendix 7.¹⁸

¹⁷Beliefs about $D_{ii}$ usually follow some justice principles. For a wider discussion review Rodriguez-Lara and Moreno-Garrido (2012a).

¹⁸The equilibrium in the $(\delta_1,\delta_2)$-ultimatum game in the general case is very laborious, because in this game, contrary to the ultimatum game, proposer does not find necessarily her maximum utility when responder accepts the offer, and it is necessary to compute payoffs when proposer wants her offer accepted or rejected. For instance, if $\delta_1$ is relatively large, then the punishment is low for proposer and she could prefer her offer be rejected.
4.1 Dictator Game

The strategic space of the dictator is $S_1 = [0, M]$. Classical model predicts that dictator should keep the whole pie. However, FS show that inequality aversion yields positive allocations to responders. This section aim is to explain how, the relative injustice aversion predicts that some dictators will pass some positive amount of money away. If the dictator (player 1) offers $x_1$ to recipient (player 2), their monetary payoffs are $p_1(x_1, x_2) = M - x_1$ and $p_2(x_1, x_2) = x_1$, respectively. Using the fact that $D_{12} = M - D_{11}$, equation (3) can be represented as

$$u_1(x_1) = M - x_1 - 2\lambda_1 \max \{x_1 - D_{12}, 0\} - 2\mu_1 \max \{D_{12} - x_1, 0\}.$$  

Since  

$$u_1(D_{12}) = M - x_1 = D_{11},$$  

the strategy $x_1 = D_{12}$ dominates the strategy of giving $x_1 > D_{12}$, because  

$$u_1(x_1) = M - x_1 - 2\lambda_1(D_{12} - x_1) < M - x_1 < D_{11} \quad \forall \ x_1 > D_{12}.$$  

Since dictator is not going to damage herself, the domain is restricted to $x_1 \in [0, D_{12}]$, dictator's utility can written as  

$$u_1(x_1) = M - x_1 - 2\mu_1(D_{12} - x_1) = M - 2\mu_1D_{12} - x_1(1 - 2\mu_1).$$  

Therefore, the theoretical prediction for dictator game is  

$$x^*_d(D_{12}) = \begin{cases} 
0 & \text{if } \mu_1 < 1/2 \\
[0, D_{12}] & \text{if } \mu_1 = 1/2 \\
D_{12} & \text{if } \mu_1 > 1/2 
\end{cases}.$$  

Note that this prediction is the same as in FS.\(^{19}\)

4.2 Ultimatum Game

The strategy space of the proposer is $S_1 = [0, M]$, and the strategy space of responder is limited to a threshold space, where responder has to set the minimum accepted offer. The strategy space of responder is an action for each information set, that is, for each $x_1 \in S_1$, responder has to decide if she accepts or rejects the offer. Hence, $S_2 : S_1 \to \{A, R\} = \{A, R\}^{[0, M]}$. In other words, responder chooses a threshold $x_2$ between 0 and $M$, such that, if $x_1 \geq x_2$, she will accept the offer and otherwise she will reject it. Therefore, with this assumption, the strategy space of both agents is the same, $S_1 = S_2 = [0, M]$.

\(^{19}\)A nonlinear version of the model would give us solutions between 0 and $D_{12}$.
For the sake of simplicity, we define for $i = 1, 2$

$$u^a_i(x_1) = u_i(x_1, x_2) \quad \text{if } x_1 \geq x_2,$$

$$u^b_i(x_1) = u_i(x_1, x_2) \quad \text{if } x_1 < x_2,$$

that represents the agent $i$’s utility function when responder accepts ($u^a_i(x_1)$) and when responder rejects ($u^b_i(x_1)$) the offer $x_1$.

Given a strategy profile $(x_1, x_2) \in S_1 \times S_2$ the material payoffs are

$$p_1(x_1, x_2) = \begin{cases} M - x_1 & \text{if } x_1 \geq x_2 \\ 0 & \text{if } x_1 < x_2 \end{cases},$$

and

$$p_2(x_1, x_2) = \begin{cases} x_1 & \text{if } x_1 \geq x_2 \\ 0 & \text{if } x_1 < x_2 \end{cases}.$$ 

Therefore, the utility functions are written (3) as

$$u^a_1(x_1) = M - x_1 - 2\lambda_1 \max \{x_1 - D_{12}, 0\} - 2\mu_1 \max \{D_{12} - x_1, 0\},$$

$$u^b_1(x_1) = -\lambda_1 \max \{D_{11} - D_{12}, 0\} - \mu_1 \max \{D_{12} - D_{11}, 0\},$$

$$u^a_2(x_1) = x_1 - 2\lambda_2 \max \{D_{22} - x_1, 0\} - 2\mu_2 \max \{x_1 - D_{22}, 0\},$$

$$u^b_2(x_1) = -\lambda_2 \max \{D_{22} - D_{21}, 0\} - 2\mu_2 \max \{D_{21} - D_{22}, 0\}.$$ 

Note that when $D_{11}$ and $D_{12}$ appears in the utility function of proposer, they stand for what proposer thinks she deserves ($D_{11}$) and what proposer thinks that responder deserves ($D_{12}$). However, when $D_{21}$ and $D_{22}$ appears in the utility function of $u_2$, they stand for what responder thinks proposer deserves ($D_{21}$) and what she thinks she deserves ($D_{22}$). In equilibrium, beliefs have to be consistent, then for the sake of simplicity, I will suppose that $D_i$ is what agent $i$ deserves. Hence, I restrict my attention to the cases where they are indeed consistent. 20 Therefore, I set $D_1 = D_{11} = D_{21}$ and $D_2 = D_{12} = D_{22}$.

Let us find the SPNE by backward induction. Given a proposal $x_1 = x^*_u$, responder has to choose whether accept or reject the offer. The best-response of responder will be a function of $M$ (the total amount), $x^*_u$ (the proposal), $D_2$ (what she deserves) and the parameters that captures the relative injustice aversion $\lambda_2$ (disadvantageous) and $\mu_2$ (advantageous).

The optimal strategy for proposer is to offer (see Appendix for details)

$$x^*_u(D_2) = \begin{cases} 0 & \text{if } \mu_1 < 1/2, \ D_2 < \frac{\mu_2}{2(\lambda_2 + \mu_2)} M \\ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{1 + 2\lambda_2} & \text{if } \mu_1 < 1/2, \ \frac{\mu_2}{2(\lambda_2 + \mu_2)} M < D_2 < M/2 \\ \frac{\lambda_2 M}{1 + 2\lambda_2} & \text{if } \mu_1 < 1/2, \ D_2 > M/2 \\ 1 & \text{if } \mu_1 > 1/2 \end{cases}.$$ 

20In lab experiments, if properties rights are not well defined (for instance where reward level are heterogeneous), the sum of what proposer thinks she deserves and what responder thinks she deserves could be different to the total amount.
or in a compact form

\[ x_u^* (D_2) = \begin{cases} 
\min \left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}, 0 \right\} & \text{if } \mu_1 < \frac{1}{2} \\
\frac{\lambda_2M}{1 + 2\lambda_2} & \text{if } \mu_1 > \frac{1}{2}
\end{cases} \]

and the responder’s minimum accepted offer is

\[ MAO_u = \min \left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}, 0 \right\} \cdot \frac{\lambda_2M}{1 + 2\lambda_2} \]

Note that \( x_u^* \) is a continuous function, except at \( \mu_1 = 1/2 \) (as in the dictator game). In any case, responder will accept the offer \( x_u^* \). When \( \mu_1 < 1/2 \), proposer exploits her bargaining power, but not totally. As it is observed in the figure 3, the amount offered, \( x_u^* \), depends on \( D_2 \). When \( D_2 \in \left( \frac{\mu_2}{2(\lambda_2 + \mu_2)}, M/2 \right) \), then \( x_u^* \) is an increasing function of \( D_2 \) with slope \( \frac{2(\lambda_2 + \mu_2)}{1 + 2\lambda_2} \) that increases with \( \mu_2 \). If \( D_2 > M/2 \), then offer does not depend on \( \mu_2 \) (because advantageous relative injustice aversion is not possible), and its value \( \frac{\lambda_2M}{1 + 2\lambda_2} \) is an increasing function of \( \lambda_2 \). In words, the larger is the responder’s disadvantageous relative injustice aversion \( \lambda_2 \), the more likely is a responder’s rejection and therefore, proposer has to offer a larger amount \( x_u^* \), in order to avoid the rejection.

It is noteworthy that since \( \mu_2 \leq \lambda_2 \), then \( \frac{\mu_2}{2(\lambda_2 + \mu_2)} \leq 1/4 \), and therefore, if \( D_2 > M/4 \), then proposer will offer a positive proportion of the pie. Note also that the interval \( \left( \frac{\mu_2}{2(\lambda_2 + \mu_2)}, M/2 \right) \), is not empty, except when if \( \lambda_2 = 0 \). In this case, since \( \mu_2 \leq \lambda_2 \), then \( \mu_2 = \lambda_2 = 0 \) and responder behaves like in the classical model, accepting any offer, i.e., \( x_u^* = 0 \).

Proposer’s bargaining power can be defined for \( \mu_1 < 1/2 \) as

\[ BP_u (D_2) = D_2 - x_u^* = \begin{cases} 
D_2 & \text{if } D_2 < \frac{\mu_2}{2(\lambda_2 + \mu_2)} \\
\frac{(1-2\mu_2)D_2 + \mu_2M}{1+2\lambda_2} & \text{if } \frac{\mu_2}{2(\lambda_2 + \mu_2)} < D_2 < D_2 < M/2 \\
\frac{\mu_2}{2(\lambda_2 + \mu_2)} + \frac{\lambda_2M}{1 + 2\lambda_2} & \text{if } D_2 > M/2
\end{cases} \]

---

21In this case, the value of \( x_u^* \) is an interval of the form \( [A, D_2] \), where \( A = 0 \) if \( D_2 < \frac{\mu_2}{2(\lambda_2 + \mu_2)} \), \( A = \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{2(\lambda_2 + \mu_2)} \) if \( \frac{\mu_2}{2(\lambda_2 + \mu_2)} < D_2 < M/2 \) and \( A = \frac{\lambda_2M}{1 + 2\lambda_2} \) if \( D_2 > M/2 \). Since \( x_u^* \) is a multivalued function is not a function but a upper hemiconcave correspondence.

22If \( \lambda_2 \) is bounded above, \( \lambda_2 \leq 1 \), then \( \frac{\lambda_2M}{1 + 2\lambda_2} \leq \frac{M}{2} \). In any case \( \frac{\lambda_2M}{1 + 2\lambda_2} \leq \frac{M}{2} \), therefore, an offer of \( x_1 = \frac{M}{2} \) should be always accepted by the responder.

23If \( \mu_1 > \frac{1}{2} \), then \( x_u^* = D_2 \), and hence for convention \( BP_u = 0 \).
Figure 3: Optimal proposal $x_u^*$ for $\mu_2 = 1/4$ (solid line), $\mu_2 = 1/2$ (dashed line) and $\mu_2 = 1$ (dotted line). Fixed values $\mu_1 < 1/2$, $\lambda_2 = 1$, $M = 1$.

Note that only when $D_2 \in \left(\frac{\mu_2}{2(\lambda_2 + \mu_2)} M, M/2\right)$, $BP_u$ depends on $\mu_2$, and in this interval, the higher $\mu_2$, the larger is proposer’s bargaining power, given that responder is less likely to reject. However, the higher $\lambda_2$, the smaller proposer’s bargaining power. In figure 4, the dependence of the bargaining power as a function of $D_2$ is shown.

Figure 4: Proposer’s bargaining power $BP_u$ for $\mu_2 = 1/4$ (solid line), $\mu_2 = 1/2$ (dashed line) and $\mu_2 = 1$ (dotted line). Fixed values $\mu_1 < 1/2$, $\lambda_2 = 1$, $M = 1$.

4.3 (0, 1)-Ultimatum Game

In the (0,1)-ultimatum game, the optimal proposer’s strategy is

$$x_{01}^* (D_2) = \begin{cases} \max \left\{ 0, \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{\mu_2 + 2\lambda_2} \right\} & \text{if } \mu_1 < \frac{1}{2} \\ \frac{D_2}{2} & \text{if } \mu_1 > \frac{1}{2} \end{cases}$$
and the responder’s minimum accepted offer is

\[ MAO_{01} = \max \left\{ \frac{2(\lambda_2 + \mu_2) D_2 - \mu_2 M}{\mu_2 + 2\lambda_2}, 0 \right\}. \]

Figure 5 represents the amount offered \( x_{01}^* \), that depends lineally on \( D_2 \), when \( D_2 \) is relatively large.

![Figure 5: Optimal proposal \( x_{01}^* \) for \( \mu_2 = 1/4 \) (solid line), \( \mu_2 = 1/2 \) (dashed line) and \( \mu_2 = 1 \) (dotted line). Fixed values \( \mu_1 < 1/2, \lambda_2 = 1, M = 1 \).](image)

Note that, even in this game, although the proposer’s bargaining power has been greatly reduced, she can exploit partially her bargaining power when \( \mu_1 < 1/2 \), offering

\[ x_{01}^* (D_2) = \max \left\{ 0, \frac{2(\lambda_2 + \mu_2) D_2 - \mu_2 M}{\mu_2 + 2\lambda_2} \right\} = \max \left\{ 0, D_2 - \frac{\mu_2 (M - D_2)}{\mu_2 + 2\lambda_2} \right\} \]

that is smaller than \( D_2 \). The bargaining power is defined as

\[ BP_{01} (D_2) = D_2 - x_{01}^* = \begin{cases} D_2 & \text{if } D_2 < \frac{\mu_2}{2(\lambda_2 + \mu_2)} M \\ \frac{\mu_2 (M - D_2)}{\mu_2 + 2\lambda_2} & \text{if } D_2 > \frac{\mu_2}{2(\lambda_2 + \mu_2)} M \end{cases} \]

As it is observed in figure 6, \( BP_{01} (D_2) \) is an increasing function to respect to \( D_2 \), until \( D_2 < \frac{\mu_2}{2(\lambda_2 + \mu_2)} M \), and it is a decreasing function respect to \( D_2 \), for \( D_2 > \frac{\mu_2}{2(\lambda_2 + \mu_2)} M \). In fact, when responder deserves the whole pie, then \( x_{01}^* (M) = M \), i.e., proposer loses the bargaining power and proposer offers the total amount to responder. In the latter interval, proposer’s bargaining powers is an increasing function respect to responder’s advantageous relative injustice aversion (\( \mu_2 \)) and a decreasing function of responder’s disadvantageous relative injustice aversion (\( \lambda_2 \)). In this particular game, a rejection is relatively more unjust for proposer than for responder. If responder does not like advantageous relative injustice aversion (\( \mu_2 \) high), then proposer has a higher bargaining power. On the other hand, if responder does not like disadvantageous relative injustice aversion (\( \lambda_2 \) high), then she is more likely to reject and hence proposer losses bargaining power.
4.4 Comparison of equilibria

In this section, the theoretical predictions of the dictator game, ultimatum game and (0, 1)-ultimatum game are compared. Observed differences will allow us to establish hypothesis about the behavior in the lab.

Dictator Game vs Ultimatum Game

The classical prediction for those games ($\mu_1 = \mu_2 = \lambda_2 = 0$) for proposer is to offer $x^* = 0$, in both games. This prediction holds in the dictator game when dictator’s advantageous relative injustice aversion is relatively small ($\mu_1 < 1/2$). However in the ultimatum game, proposer cannot exploit all her bargaining power, because according to responder’s parameters $\mu_2$ and $\lambda_2$, she could reject the offer. In any case, if proposer’s advantageous relative injustice aversion is relatively large ($\mu_1 > 1/2$), either in dictator game or ultimatum game, proposers do not want to exploit her bargaining power and she offers $x^* = D_2$.

When $\mu_1 < 1/2$, then $x^*_d = 0$, and the excess of proposer’s bargaining power in the dictator game respect to the ultimatum game is computed as

$$BP_d - BP_u = (D_2 - x^*_d) - (D_2 - x^*_u) = x^*_u \geq 0.$$  

Therefore, as in classical model of utility function, it is expected a greater offer in the ultimatum game than in the dictator game.

Ultimatum Game vs (0, 1)-Ultimatum Game

Proposers

The classical prediction for ultimatum game ($\mu_1 = 0$) for proposer is to offer $x^*_u = 0$. However, for the (0, 1)-ultimatum game, it is found multiple equilibria.
Our predictions with the relative injustice aversion utility function are

\[ x_u^* = \begin{cases} 
\min \left\{ \lambda^2 \frac{\mu_2}{1 + 2\lambda_2}, 0 \right\} & \text{if } \mu_1 < \frac{1}{2} \\
D_2 & \text{if } \mu_1 > \frac{1}{2} 
\end{cases} \]

for the ultimatum game and

\[ x_{01}^* = \begin{cases} 
\max \left\{ 0, \frac{\lambda^2 + \mu_2}{\mu_2 + 2\lambda_2} \right\} & \text{if } \mu_1 < \frac{1}{2} \\
D_2 & \text{if } \mu_1 > \frac{1}{2} 
\end{cases} \]

for the (0,1)UG.

Let us suppose that \(2(\lambda_2 + \mu_2)D_2 > \mu_2M\), because otherwise, \(x_u^* = x_{01}^* = 0\). If proposer’s advantageous relative injustice aversion is relatively large (\(\mu_1 > 1/2\)), then proposer does not want to exploit her bargaining power and the prediction in both games is the same (\(x_u^* = x_{01}^* = D_2\)). However, if \(\mu_1 < 1/2\), since \(\mu_2 + 2\lambda_2 < 1 + 2\lambda_2\), then, if \(2(\lambda_2 + \mu_2)D_2 > \mu_2M\), it is satisfied

\[
2(\lambda_2 + \mu_2)D_2 - \mu_2M < 2(\lambda_2 + \mu_2)D_2 - \mu_2M, \\
\]

and therefore

\[
x_u^* = \min \left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}, \frac{\lambda_2M}{1 + 2\lambda_2} \right\} < \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{\mu_2 + 2\lambda_2} = x_{01}^*. \\
\]

Hence, when \(\mu_1 < 1/2\), it is expected that proposer offers a greater amount in the (0,1)UG than in the ultimatum game. I compute the excess of proposer’s bargaining power in the ultimatum game respect to the (0,1)UG as \(BP_u - BP_{01}\).

For \(D_2 \leq M/2\), the bargaining power in the ultimatum game is greater than in the (0,1)UG if and only if \(\mu_2 \leq 1\).

\[
BP_u - BP_{01} = (1 - \mu_2) \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{(\mu_2 + 2\lambda_2)(1 + 2\lambda_2)} > 0. \\
\]

As it is observed in figure 8, when \(D_2 \leq M/2\), differences of bargaining power between the ultimatum game and the (0,1)UG goes to zero when \(\mu_2\) increases from 0 to 1.

For \(D_2 \geq M/2\), \(x_u^* = \frac{\lambda_2M}{1 + 2\lambda_2}\), and \(BP_u = D_2 - \frac{\lambda_2M}{1 + 2\lambda_2}\), and therefore

\[
BP_u - BP_{01} = \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{\mu_2 + 2\lambda_2} - \frac{\lambda_2M}{1 + 2\lambda_2} = \frac{(\lambda_2 + \mu_2)(2D_2 - M)}{\mu_2 + 2\lambda_2} > 0. \\
\]
Figure 7: Comparison of optimal proposals $x_u^*$ (black) and $x_{01}^*$ (gray) for $\mu_2 = 1/4$ (solid line), $\mu_2 = 1/2$ (dashed line) and $\mu_2 = 1$ (dotted line). Fixed values $\mu_1 < 1/2$, $\lambda_2 = 1$, $M = 1$.

Figure 8: Comparison of proposer's bargaining power $BP_u$ (black) and $BP_{01}$ (gray) for $\mu_2 = 1/4$ (solid line), $\mu_2 = 1/2$ (dashed line) and $\mu_2 = 1$ (dotted line). Fixed values $\mu_1 < 1/2$, $\lambda_2 = 1$, $M = 1$.

Responders

The classical prediction for ultimatum game ($\mu_1 = 0$) for responders, is that the minimum accepted offer is $MAO_u = 0$. However, for the $(0, 1)$UG, any minimum accepted offer can be part of the equilibria. In the current model, if $\mu_2 \leq 1$,\(^\text{25}\) then $\mu_2 + 2\lambda_2 \leq 1 + 2\lambda_2$, and then

$$\frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{\mu_2 + 2\lambda_2} \geq \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{1 + 2\lambda_2}$$

and therefore $MAO_{01} \geq MAO_u$.

5 Conclusions

I present a utility function model with the same philosophy than FS, that is able to explain rejections in non zero-sum games with production. The relative injustice aversion utility function represents the same preferences than the inequality aversion utility function when

\(^{24}\)If responder’s advantageous relative injustice aversion is greater than 1 ($\mu_2 > 1$), it is possible to find offers such that the responder would reject in the ultimatum game, but not in the $(0, 1)$UG. It suggests that $\mu_2$ should be bounded above by 1.

\(^{25}\)The upper bound of $\mu_1$ is justified in the footnote 24.
game is a zero-sum game, and the justice principle followed is the egalitarian principle. The contribution of the new utility function is the possibility of defining a justice point, that depends of agent’s contribution and on agent’s believes according to some justice principle.

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References


6 Appendix

6.1 Lemma

Let be the following variables

\[ T_3 = \frac{2(\lambda_2 + \mu_2) D_2 - \mu_2(1 - \delta_1)M}{1 + 2\lambda_2 - \delta_2 + \mu_2(\delta_1 + \delta_2)}, \quad T_4 = \frac{\lambda_2(1 - \delta_1)M}{\lambda_2(1 - \delta_1) + (1 + \lambda_2)(1 - \delta_2)}, \]

\[ C_{34} = \frac{2D_2 - M(1 - \delta_1)}{(\delta_1 + \delta_2)}, \quad D^* = \frac{M}{2} \frac{(1 - \delta_1)(1 + 2\lambda_2 - \delta_2)}{1 + 2\lambda_2 - \delta_2 - \lambda_2(\delta_1 + \delta_2)}, \]
Therefore, the lemma is rewritten as
\[ D_T D > D \] and solving for
\[ \text{If } (\delta_1, \delta_2) \neq (0, 0). \]
From the inequalities i) \( T_3 < C_{34} \), ii) \( T_4 < C_{34} \) and iii) \( T_4 < T_3 \), and solving for \( D_2 \), it is obtained that \( D_2 > D^* \) in all the cases.

In the case of the ultimatum game \( (\delta_1 = \delta_2 = 0) \), the variables are redefined as
\[ T_3 = \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{1 + 2\lambda_2}, \quad T_4 = \frac{\lambda_2 M}{1 + 2\lambda_2}, \quad C_{34} = \frac{2D_2 - M}{\lambda} = \left\{ \begin{array}{ll} +\infty & \text{if } D_2 > M/2 \\ -\infty & \text{if } D_2 < M/2 \end{array} \right\}, \quad D^* = \frac{M}{2}. \]
Therefore, the lemma is rewritten as \( D_2 > M/2 \Leftrightarrow \frac{\lambda_2 M}{1 + 2\lambda_2} < \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{1 + 2\lambda_2} < +\infty \) and the prove is trivial.

6.2 Ultimatum Game

Since it is supposed that \( D_1 = D_2 = D_1 \) and \( D_1 = D_2 = D_2 \), and agents are rational, then \( D_1 + D_2 = M \), therefore the utility functions can be written as a function of \( D_2 \)

\[
\begin{align*}
u_1^a(x_1) &= M - x_1 - 2\lambda_1 \max\{ x_1 - D_2, 0 \} - 2\mu_1 \max\{ D_2 - x_1, 0 \}, \\
u_1^r(x_1) &= -2\lambda_1 \max\{ M/2 - D_2, 0 \} - 2\mu_1 \max\{ D_2 - M/2, 0 \}, \\
u_2^a(x_1) &= x_1 - 2\lambda_2 \max\{ D_2 - x_1, 0 \} - 2\mu_2 \max\{ x_1 - D_2, 0 \}, \\
u_2^r(x_1) &= -2\lambda_2 \max\{ D_2 - M/2, 0 \} - 2\mu_2 \max\{ M/2 - D_2, 0 \}.
\end{align*}
\]

Responder’s strategy

When responder accepts the offer, what determines the envy/guilty feeling is whether \( D_2 \) is greater or smaller than \( x_1 \). Contrary, if she rejects the offer, what determines the feeling is whether \( 2D_2 - M \) is greater or smaller than zero.

SPNE are studied using backward induction. Four possible cases are studied, denoted as \( (A, R) \), where \( A, R \in \{ \lambda, \mu \} \). The four cases are summarized as \( (A, R) \), where

\[
A = \begin{cases} 
\mu & \text{if } x^* > D_2 \\
\lambda & \text{if } x^* < D_2
\end{cases}
\quad \text{and} \quad
R = \begin{cases} 
\mu & \text{if } M/2 > D_2 \\
\lambda & \text{if } M/2 < D_2
\end{cases}
\]

where \( \lambda \) stands for envy and \( \mu \) for guilty.

**Case 1:** \( (\mu, \mu) \). She deserves less than the half of the pie \( (M/2 > D_2) \) and the offer is greater than what she deserves \( (x^* > D_2) \). She will feel guilty in any case, whether she accepts or if she rejects the offer.
If responder accepts, her utility is $u^2_f(x^*) = x^* - 2\mu_2(x^* - D_2)$. If she rejects, her utility is $u^2_r(x^*) = -\mu_2(M - 2D_2)$. She will accept the offer if $x^* - 2\mu_2(x^* - D_2) > -\mu_2(M - 2D_2)$, but the latter equation is true because it is equivalent to $x^*(1 - \mu_2) + \mu_2(M - x^*) > 0$.

**Case 2:** $(\mu, \lambda)$. She will feel guilty if she accepts the offer and she will feel envy if she refuses the offer. If responder accepts, her utility is $u^2_f(x^*) = x^* - 2\mu_2(x^* - D_2)$ and if she rejects the offer, her utility is $u^2_r(x^*) = -\mu_2(2D_2 - M)$.

She will accept if $x^* - 2\mu_2(x^* - D_2) > -\lambda_2(2D_2 - M)$, that is equivalent to $x^* - 2\mu_2(x^* - D_2) + \lambda_2(2D_2 - M) > 0$, that is true because by assumption $2D_2 - M > 0$, and then $x^* < 2D_2$.

Cases 1 and 2 can be summed up as the fact that responder will accept any offer $x^* \geq D_2$.

**Case 3:** $(\lambda, \mu)$. She will feel envy if she accepts the offer ($D_2 > x^*$) and she will feel guilty if she refuse the offer ($M/2 > D_2$). If she accepts, her utility is $u^3_f(x^*) = x^* - \lambda_2(2D_2 - 2x^*)$, and if she rejects the offer, her utility is $u^3_r(x^*) = -\mu_2(M - 2D_2)$. She will accept if $x^* - \lambda_2(2D_2 - 2x^*) \geq -\mu_2(M - 2D_2)$, that is, when $x^* \geq \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}$. We call $t_3 = \max\left\{\frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}, 0\right\}$.

**Case 4:** $(\lambda, \lambda)$. She deserves more than the half of the pie ($D_2 > M/2$) and the offer is smaller than what she deserves ($D_2 > x^*$). If she accepts, her utility is $u^4_f(x^*) = x^* - \lambda_2(2D_2 - 2x^*)$ and if she rejects the offer, her utility is $u^4_r(x^*) = -\lambda_2(2D_2 - M)$. She will accept the offer if $x^* \geq \frac{\lambda_2M}{1 + 2\lambda_2}$. We call $T_4 = \frac{\lambda_2M}{1 + 2\lambda_2}$, that is the minimum amount accepted offer.

Responder’s strategy can be summarized as follows. If $x^* \geq D_2$, then responder accepts, otherwise, $x^* < D_2$, if $D_2 < M/2$, responder will accept if $x^* \geq \max\left\{\frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}, 0\right\}$. However, if $D_2 > M/2$, then responder will accept if $x^* \geq \frac{\lambda_2M}{1 + 2\lambda_2}$.

**Proposer’s strategy**

If responder accepts the offer, proposer’s utility is

$$u^a_1(x^*) = M - x^* - \lambda_1 \max\{2x^* - 2D_2, 0\} - \mu_1 \max\{2D_2 - 2x^*, 0\},$$

and if responder rejects the offer

$$u^a_1(x^*) = -\lambda_1 \max\{M - 2D_2, 0\} - \mu_1 \max\{2D_2 - M, 0\}.$$

---

26 For the sake of simplicity we call $T_3 = \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{1 + 2\lambda_2}$ and $t_3 = \max\{T_3, 0\}$. Note that $T_3 < D_2$, and therefore an offer of $x^* = D_2$ is always accepted. However, an offer of $x^* = 0$, is accepted if and only if $D_2 < \frac{\mu_2M}{\lambda_2 + \mu_2}$. Since $\frac{\mu_2M}{\lambda_2 + \mu_2} \in [0, M/4]$, then, if $D_2 > M/4$, then responder will reject a null offer.

27 Note that $T_4 > 0$. Under the assumption $D_2 > x^*$, it holds that $T_4 < D_2$. Note also that if $x^* = M/2$, then responder will accept, therefore, although responder deserves more than the half of the pie, she will accept $x^* = M/2$, because a rejection would cause to responder a greater disadvantageous injustice to herself than to proposer, because responder’s deserving is greater that proposer’s deserving.
The strategy of proposer will be to offer \( x^* \) such that she accepts, because \( x_1 = D_2 \) strongly dominates \( x_1 = x^* \), such that \( x^* \) will be rejected by responder because \( u_1^*(D_2) = M - D_2 = D_1 \geq 0 \), and \( u_1^*(x) < 0 \). If \( D_2 \) would be unknown by proposer, then the acceptance can be assured, by offering \( x^* = M/2 \) (see cases 3 and 4 of responder’s strategy), because \( u_1^*(M/2) > u_1^*(x) \). The aim of proposer is to consider if she can offer an amount \( x^* < \min\{D_2, M/2\} \) that will be accepted by responder and in this way to exploit her bargaining power. Thus, only cases 3 and 4 of strategy of responder are considered.

If \( D_2 < M/2 \) (case 3), then responder will accept if \( x^* \in [t_3, D_2] \) and she will reject if \( x^* \in [0, t_3) \). Let us compute the critical points in the interior of the interval \( x_1 \in [t_3, D_2] \) (when responder accepts). Since \( u_1^*(x_1) = M - x_1 - 2\mu_1(D_2 - x_1) \), then \( \frac{du_1^*}{dx_1}(x_1) = -1 + 2\mu_1 \).

Hence, if \( \mu_1 > 1/2 \) then \( \frac{du_1^*}{dx_1}(x_1) > 0 \) and \( x^* = D_2 \). However, if \( \mu_1 < 1/2 \) then \( \frac{du_1^*}{dx_1}(x_1) < 0 \) and \( x^* = \min\{t_3, T_4\} = t_3.28 \)

If \( D_2 > M/2 \) (case 4), then, responder will accept if \( x^* \in [T_4, D_2] \) and she will reject if \( x^* \in [0, T_4) \). The minimum acceptable offer by responder is \( x^* = \min\{t_3, T_4\} = T_4 \). Since \( u_1^*(x^*) = M - x^* - 2\lambda_2(x^* - D_2) \), then \( \frac{du_1^*}{dx_1}(x_1) = -1 - 2\lambda_1 < 0 \), and therefore, proposer will offer the possible minimum in the interval \([T_4, D_2]\), i.e., \( x^* = T_4 \).

**Equilibrium**

The optimal strategy for proposer is to offer\(^{29}\)

\[
x^* = \min \left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{1 + 2\lambda_2}, \frac{\lambda_2 M}{1 + 2\lambda_2} \right\},
\]

if \( \mu_1 < 1/2 \).

The optimal strategy for responder is to accept any

\[
x^* \geq \min \left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{1 + 2\lambda_2}, \frac{\lambda_2 M}{1 + 2\lambda_2} \right\},
\]

and reject any another offer.

### 6.3 (0, 1)-Ultimatum Game

With the same assumptions than in the ultimatum game, the utility function of proposer and responder is written as a function of \( D_2 \)

\[
u_1^*(x_s) = M - x_1 - 2\lambda_1 \max \{x_t - D_2, 0\} - 2\mu_1 \max \{D_2 - x_1, 0\},
\]

\[
u_1^*(x_1) = -\lambda_1 \max \{M - 2D_2 + x_1, 0\} - \mu_1 \max \{2D_2 - M - x_1, 0\},
\]

\(^{28}\)In the ultimatum game, \( D^* = M/2 \), therefore \( D_2 < M/2 \) implies that \( T_3 < T_4 \), and \( D_2 > M/2 \) implies that \( T_3 > T_4 \).

\(^{29}\)Note, that if \( D_2 < \frac{\mu_2}{\lambda_2 + \mu_2} \frac{M}{2} \), then \( x^* = 0 \).
\[ u_2^b(x_\ast) = x_1 - 2\lambda_2 \max\{D_2 - x_\ast, 0\} - 2\mu_2 \max\{x_1 - D_2, 0\}, \]
\[ u_2^r(x_1) = x_1 - \lambda_2 \max\{2D_2 - M - x_1, 0\} - \mu_2 \max\{M - 2D_2 + x_1, 0\}. \]

SPNE are studied using backward induction with the same four cases than in the ultimatum game depending if responder feels envy or guilty when she accepts or rejects the offer.

**Responder’s decision**

If responder accepts the offer, what determines the envy/guilty feeling is whether \( D_2 \) is greater or smaller than \( x_1 \). Contrary, if she rejects the offer, what determines the feeling is whether \( 2D_2 - M - x_1 \) is greater or smaller than zero, that is equivalent to the fact that \( x_1 \) is greater or smaller than \( 2D_2 - M \).

**Case 1:** \((\mu, \mu)\). \( x^\ast > D_2 \) and \( x^\ast > 2D_2 - M \). She will have envy in any case. If she accepts, her utility is \( u_2^b(x^\ast) = x^\ast - 2\mu_2(x^\ast - D_2) \), and if she rejects the offer, her utility is \( u_2^r(x^\ast) = x^\ast - \mu_2(x^\ast + M - 2D_2) \). She will accept because \( u_2^b(x^\ast) \geq u_2^r(x^\ast) \) is equivalent to \( x^\ast \leq M \).

**Case 2:** \((\mu, \lambda)\). \( x^\ast > D_2 \) and \( x^\ast < 2D_2 - M \). She will feel guilty if she accepts the offer, because \( x^\ast \) is greater than she deserves. However, she will feel envy if she refuses the offer.

Assumptions implies that \( x^\ast = D_2 = M \), and in this case, responder is indifferent between accepting or rejecting the offer, because \( u_2^b(D_2) = D_2 \) and \( u_2^r(D_2) = D_2 - \lambda_2(D_2 - M) = D_2 \).

**Case 3:** \((\lambda, \mu)\). \( x^\ast < D_2 \) and \( x^\ast > 2D_2 - M \). She will feel envy if she accepts the offer, because \( x^\ast \) is smaller than she deserves. However, she will feel guilty if she refuses the offer.

If she accepts, her utility is \( u_2^b(x^\ast) = x^\ast - \lambda_2(2D_2 - 2x^\ast) \), and if she rejects the offer, her utility is \( u_2^r(x^\ast) = x^\ast - \mu_2(x^\ast + M - 2D_2) \). Thus, she will accept the offer if and only if \( x^\ast \geq \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{2\lambda_2 + \mu_2} \). We call \( t_3 = \max\left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{2\lambda_2 + \mu_2}, 0\right\} \). For the sake of simplicity we call \( T_3 = \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{2\lambda_2 + \mu_2} \) and \( t_3 = \max\{t_3, 0\} \).

**Case 4:** \((\lambda, \lambda)\). \( x^\ast \leq D_2 \) and \( x^\ast \leq 2D_2 - M \). She will feel envy in any case. If she accepts, her utility is \( u_2^b(x^\ast) = x^\ast - \lambda_2(2D_2 - 2x^\ast) \), and if she rejects the offer, her utility is \( u_2^r(x^\ast) = x^\ast - \lambda_2(2D_2 - M - x^\ast) \). Thus, she will accept the offer if and only if \( x^\ast \geq M \).

Cases 1 to 4 can be summarized as follow: If \( x^\ast \geq D_2 \), then responder accepts, otherwise, if \( x^\ast < D_2 \) then if \( x^\ast \geq 2D_2 - M \), responder will accept if \( x^\ast \geq \max\left\{ \frac{2(\lambda_2 + \mu_2)D_2 - \mu_2 M}{2\lambda_2 + \mu_2}, 0\right\} \).

However, if \( x^\ast \leq 2D_2 - M \), responder will accept only if \( x^\ast = M \). Since \( T_3 \geq 2D_2 - M \), then, we can write responder’s strategy as follows: if \( x^\ast \geq D_2 \), then responder accepts. If \( x^\ast < D_2 \) then the minimum amount offer is \( x^\ast = t_3 \).

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Note that \( t_3 < D_2 \), and therefore, an offer of \( x^\ast = D_2 \) is always accepted.

Note that this case, is only possible when \( D_2 = M \).
Proposer’s decision

From the point of view of proposer she has to chose $x_*$ such that maximizes her payoff. If proposer offers $x^* = D_2$, then responder will accept and the proposer’s payoff is $u_1^q(D_2) = (M - D_2) = D_1$. If proposer offer $x^* > D_2$, responder will accept and proposer’s utility is $u_1^q(x^*) = M - x^* - 2\lambda_1(x_1 - D_2) < M - x^* < M - D_2 = D_1$. The strategy of offering $x^* > D_2$ is dominated by $x_1 = D_2$, and the strategy of offering $x^* < T_3$ is also dominated by $x_1 = D_2$.\footnote{If proposer offers $x_* < t_3$, responder rejects the offer and hence proposer’s utility is negative $u_1^q(x_1) = -\lambda_1\max\{M - 2D_2 + x_1, 0\} - \mu_1\max\{2D_2 - M - x_1, 0\} < 0 \ \forall x_1 \in [0, T_3)$.}

We only need to study the utility function $u_1^q(x_1)$ in the interval $[t_3, D_2]$. Since $u_1^q(x_1) = M - x_1 - 2\mu_1(D_2 - x_1) \ \forall x_1 \in [T_3, D_2]$, its derivative respect to $x_1$, is $\frac{du_1^q}{dx_1}(x_1) = -1 + 2\mu_1$, and therefore, if $\mu_1 > 1/2$ then $x^* = D_2$ and if $\mu_1 < 1/2$, then $x^* = t_3$.

Equilibrium

The best strategy for proposer is to offer $x_{01}^*$, such that, if $\mu_1 > 1/2$, then $x_{01}^* = D_2$ and if $\mu_1 < 1/2$, then $x_{01}^* = T_3$, and the minimum amount that responder is willing to accepts is $t_3 = \max\left\{\frac{2(\lambda_2 + \mu_2)D_2 - \mu_2M}{2\lambda_2 + \mu_2}, 0\right\}$.\footnote{If proposer offers $x_* < t_3$, responder rejects the offer and hence proposer’s utility is negative $u_1^q(x_1) = -\lambda_1\max\{M - 2D_2 + x_1, 0\} - \mu_1\max\{2D_2 - M - x_1, 0\} < 0 \ \forall x_1 \in [0, T_3)$.}
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